CONTINUING EDUCATION Mathematics Department - MCV4U PRACTICE EXAM

PART A: Fill in the blanks. Write your answer on the line provided.

1. Evaluate $\lim_{x \to 2} \left(\frac{\sqrt{x+2}-2}{x-2} \right) \times \frac{\sqrt{x+2}+2}{\sqrt{x+2}+2}$ $\frac{x+\cancel{2}-\cancel{4}}{(\cancel{x-2})(\sqrt{x+2}+2)}$ $\underline{1/4}$

2. Determine the $\lim_{x\to\infty} \frac{3x^2+1}{4x^2-2}$ $\underline{3/4}$

3. What does $\lim_{h\to0} \frac{(x+h)^2-x^2}{h}$ equal? $\frac{\cancel{x^2}+2x\cancel{h}+h^2-\cancel{x^2}}{\cancel{h}}$ $\underline{2x}$

4. Does $f(x)=2x^2-8x$ have any inflection points? $4x-8$ 4 $\underline{\text{No}}$

5. For the function $f(x)=\begin{cases} x^2+3, & x\geq 2 \\ 2x+3, & -1\leq x<2 \\ x^3, & x<-1 \end{cases}$

   (a) Evaluate $f(-1)$ $-2+3$ $\underline{1}$

   (b) Evaluate $\lim_{x\to-1} f(x)$ $\underline{\text{DNE}}$

   (c) Evaluate $\lim_{x\to2} f(x)$ $\underline{7}$

6 Differentiate

   (a) $f(x)=2x^3-x$ $\underline{6x^2-1}$

   (b) $y=\frac{1}{x^4}$ $\underline{-4/x^5}$

   (c) $f(x)=5^{(2x)}$ $\underline{5^{2x}\cdot \ln 5\cdot 2}$

   (d) $f(x)=e^{\cos x}$ $\underline{e^{\cos x}(-\sin x)}$

   (e) $f(x)=\tan(3x-4)$ $\underline{3\sec^2(3x-4)}$

7. Determine $g(f(2))$ if $f(x)=\sqrt{x^2+5}$ and $g(x)=x^2+x-2$ 3^2+3-2 $\underline{10}$

8. If $y=u^3+u^2-1$, where $u=\frac{1}{1-x}$, determine $\frac{dy}{dx}$ at x = 2. $\frac{dy}{dx}=-1$

Let $f(x)=\frac{2x^3-x^2+2x+1}{x^2+1}$. What is the **equation** of the oblique asymptote. $\underline{\text{Don't do!}}$

$y=2x-1$

Part B: **Short Answer Questions**

Answer in the space provided

1. <u>*Using First Principles*</u> (ie: the limit definition of a derivative), determine the derivative of $f(x)=\frac{3}{x^2}$

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{\frac{3}{(x+h)^2}-\frac{3}{x^2}}{h}$$

$$= \lim_{h\to 0}\frac{3x^2-3(x+h)^2}{x^2(x+h)^2}\cdot\frac{1}{h}$$

$$= \lim_{h\to 0}\frac{\cancel{3x^2}-\cancel{3x^2}-6x\cancel{h}-3h^{\cancel{2}}}{x^2(x+h)^2}\cdot\frac{1}{\cancel{h}}$$

$$= \lim_{h\to 0}\frac{-6x-3h}{x^2(x+h)^2}$$

$$= \frac{-6x}{x^4}$$

$$= \frac{-6}{x^3}$$

2. Differentiate the following functions. <u>**Do Not Simplify**</u>.

a) $y=\frac{1+x^2}{1+x^3}$

$$y' = \frac{2x(1+x^3)-3x^2(1+x^2)}{(1+x^3)^2}$$

$$= \frac{2x+2x^4-3x^2-3x^4}{(1+x^3)^2}$$

b) $g(x)=(x^3-2x+3)^4$

$$g'(x) = 4(x^3-2x+3)^3(3x^2-2)$$

c) $f(x)=(x^2+1)^3(7x+2)^2$

$$f'(x) = (x^2+1)^3\,2(7x+2)(7) + (7x+2)^2(3)(x^2+1)^2(2x)$$

3. Find the equation of the line tangent to the curve USING IMPLICIT DIFFERENTIATION on $x^2 - 2xy + 8 = 0$ at the point (2,3) in ***standard form.***

$$2x - \left(2x\frac{dy}{dx} + 2y\right) = 0$$

$$2x - 2x\frac{dy}{dx} - 2y = 0$$

$$2x - 2y = 2x\frac{dy}{dx}$$

$$\frac{2x-2y}{2x} = \frac{dy}{dx}$$

$$1 - \frac{2y}{2x} = \frac{dy}{dx}$$

$$1 - \frac{2(3)}{2(2)} = \frac{dy}{dx}$$

$$1 - \frac{6}{4} = \frac{dy}{dx}$$

$$-\frac{1}{2} = \frac{dy}{dx}$$

$$y - 3 = -\frac{1}{2}(x-2)$$

$$2y - 6 = -x + 2$$

$$0 = -x - 2y + 8$$

$$\boxed{0 = x + 2y - 8}$$

4. Determine the limit of the following function $\lim\limits_{x \to 125} \dfrac{125 - x}{x^{\frac{1}{3}} - 5}$

let $x^{1/3} = u$ $\quad 125^{1/3} = u$

$x = u^3$ $\quad 5 = u$

$$\lim_{u \to 5} \frac{125 - u^3}{u - 5}$$

$$= \lim_{u \to 5} \frac{(5-u)(25 + 5u + u^2)}{u-5}$$

$$= \lim_{u \to 5} (-1)(25 + 5u + u^2)$$

$$= (-1)(25 + 5(5) + 5^2)$$

$$= -75$$

5. A train travels along a straight track in a way that its position, in meters, can be expressed as a function $s(t) = 6t^2 + 4t$ where t is in seconds.

a) Determine the average velocity between t = 3 s. and t = 6 s.
b) Determine the instantaneous velocity when t = 6 s.
c) Determine the acceleration.

a) $\frac{s(6)-s(3)}{6-3} = \frac{6(6)^2+4(6)-(6(3)^2+4(3))}{6-3}$

$= \frac{216+24-54-12}{3}$

$= 174/3$

$= 58$

b) $v(t) = 12t + 4$

$v(6) = 12(6) + 4$

$= 72 + 4$

$= 76$

c) $a(t) = 12$

Part C: **Problem Solving**

Show all your work, providing neat and clear solutions in the space provided.

1. Find and classify any turning points for the function $f(t) = 2t^3 - 21t^2 + 60t$

$f'(t) = 6t^2 - 42t + 60$

$0 = 6t^2 - 42t + 60$

$= t^2 - 7t + 10$

$= (t-5)(t-2)$

$t = 5, 2$

$f''(t) = 12t - 42$

$f''(5) = 12(5) - 42$

$= 60 - 42$

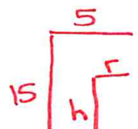
$= 18$ ∴ concave up min @ t=5

$f''(2) = 12(2) - 42$

$= 24 - 42$

$= -18$ ∴ concave down max @ t=2

2. A conical paper cup, with radius of 5 cm and height of 15 cm, is leaking water at a rate of 2 cm^3/min. At what rate is the water level decreasing when the water is 3 cm deep? [$V = \frac{1}{3}\pi r^2 h$]



$\frac{5}{r} = \frac{15}{h}$

$5h = 15r$

$h = 3r$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi r^2 (3r)$

$= \pi r^3$

$\frac{dV}{dt} = 3\pi r^2 \frac{dr}{dt}$

$2 = 3\pi (1)^2 \cdot \frac{dr}{dt}$

$\frac{2}{3\pi} = \frac{dr}{dt}$

$V = \frac{1}{3}\pi \left(\frac{h}{3}\right)^2 \cdot h$

$= \frac{1}{3}\pi \frac{h^2}{9} \cdot h$

$= \frac{\pi h^3}{27}$

$\frac{dV}{dt} = \frac{1}{9}\pi h^2 \cdot \frac{dh}{dt}$

$3 = \frac{1}{9}\pi (3)^2 \cdot \frac{dh}{dt}$

$3 = \pi \cdot \frac{dh}{dt}$

$\frac{3}{\pi} = \frac{dh}{dt}$

3. A can is to be made to hold a litre of oil. What dimensions of the can are needed to minimize the cost of the metal to make the can. [1 L = 1000 cm^3 , $V = \pi r^2 h$, $SA = 2\pi r^2 + 2\pi rh$]

$1000 = \pi r^2 h$

$\frac{1000}{\pi r^2} = h$

$SA = 2\pi r^2 + 2\pi r h$

$= 2\pi r^2 + 2\pi r \left(\frac{1000}{\pi r^2}\right)$

$= 2\pi r^2 + \frac{2000}{r}$

$SA' = 4\pi r - \frac{2000}{r^2}$

$0 = 4\pi r - \frac{2000}{r^2}$

$4\pi r^3 = 2000$

$r^3 = \frac{2000}{4\pi}$

$r^3 = \frac{500}{\pi}$

$r = \sqrt[3]{\frac{500}{\pi}}$

4. The curve defined by $f(x) = \dfrac{x^2 + 2x - 3}{x^2}$ has $f'(x) = \dfrac{-2x + 6}{x^3}$, and $f''(x) = \dfrac{4x - 18}{x^4}$.

a) State the equations of any asymptotes. Examine the end behaviours.

V.A. $x = 0$

H.A. $y = 1$

$\lim_{x \to \infty} \frac{x^2 + 2x - 3}{x^2}$

$= \lim_{x \to \pm\infty} \frac{\cancel{x^2}\left(1 + \frac{2}{x} - \frac{3}{x^2}\right)}{\cancel{x^2}}$

$= 1$

$f(1000) = \frac{1000^2 + 2(1000) - 3}{1000^2}$

$f(-1000) = \frac{(-1000)^2 + 2(-1000) - 3}{(-1000)^2}$

b) Determine the co-ordinates of the x and y intercepts.

x-int: $0 = \frac{x^2+2x-3}{x^2}$

$0 = x^2+2x-3$

$= (x+3)(x-1)$

$x = -3, 1$

y-int $y = \frac{0^2+2(0)-3}{0^2}$

$= \frac{-3}{0}$

No y-int.

c) Determine the coordinates of all maximum and minimum points. Justify your answers.

$-2x+6=0$

$2x=6$

$x=3$

$y = \frac{3^2+2(3)-3}{3^2}$

$= \frac{12}{9}$

$= \frac{4}{3}$

$f''(3) = \frac{4(3)-18}{3^4}$

$= \frac{-6}{81}$

$\therefore$ max at $(3, 4/3)$

d) Determine the *x coordinates* of all points of inflection.

$4x-18=0$

$4x=18$

$x=\frac{9}{2}$

e) Graph the function and ***identify any significant features***.